Exercise 69

In Chapter 9 we will be able to show, under certain assumptions, that the velocity $v(t)$ of a falling raindrop at time t is

$$v(t) = v^*(1 - e^{-gt/v^*})$$

where g is the acceleration due to gravity and v^* is the *terminal velocity* of the raindrop.

(a) Find $\lim_{t\to\infty} v(t)$.

(b) Graph $v(t)$ if $v^* = 1$ m/s and $g = 9.8$ m/s^2. How long does it take for the velocity of the raindrop to reach 99% of its terminal velocity?

Solution

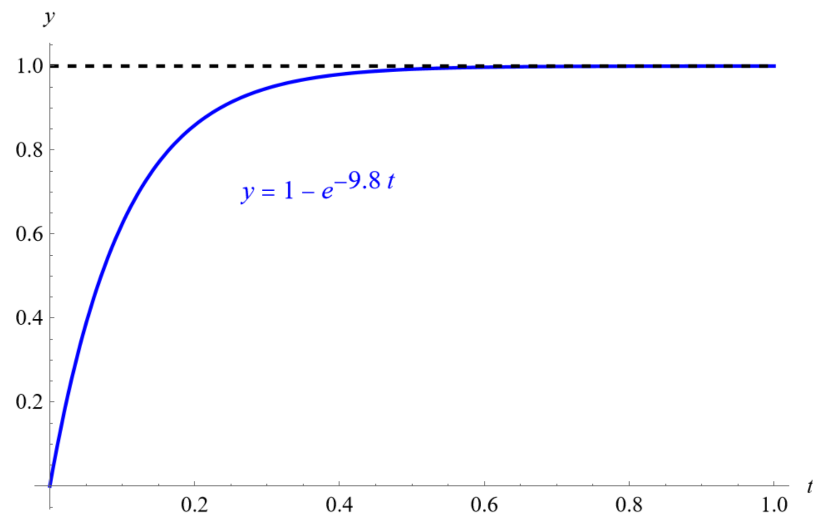
Calculate the limit of the velocity as $t \to \infty$.

$$\lim_{t\to\infty} v(t) = \lim_{t\to\infty} v^*(1 - e^{-gt/v^*}) = \lim_{t\to\infty} v^*\left(1 - \frac{1}{e^{gt/v^*}}\right) = v^*(1 - 0) = v^*$$

If $v^* = 1$ m/s and $g = 9.8$ m/s^2, then

$$v(t) = 1 - e^{-9.8t}.$$

A graph of this function versus t is shown below.



To find how long it takes for the velocity of the raindrop to reach 99% of its terminal velocity, set $v(t) = 0.99v^* = 0.99(1) = 0.99$ and solve the equation for t.

$$0.99 = 1 - e^{-9.8t}$$

$$e^{-9.8t} = 0.01$$

$$\ln e^{-9.8t} = \ln 0.01$$

$$-9.8t \ln e = \ln 0.01$$

$$t = -\frac{1}{9.8}\frac{\ln 0.01}{\ln e} \approx 0.469915 \text{ seconds}$$